World Volume Action for Fractional Branes [*]

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Abstract

We study the world volume action of fractional Dp-branes of type IIA string theory compactified on the orbifold T^4/Z_2. The geometric relation between these branes and wrapped branes is investigated using conformal techniques. In particular we examine in detail various scattering amplitudes and find that the leading low-energy interactions are consistent with the boundary action derived geometrically.

Contents

1 Introduction

Maldacena's duality [1] provides a remarkable relation between string theory and supersymmetric and conformal quantum field theory. Recently a lot of effort has been devoted to extend this duality and find new correspondences between string theories and non conformal and less supersymmetric gauge theories. These attempts have been developed in several directions, one of which is the study of fractional D-branes [2] on conifold [3] and orbifold singularities [4, 5]. In this case the dual gauge theory corresponding to a stack of M such fractional branes is non conformal and supersymmetry is partially broken with respect to the original case considered by Maldacena.

It is well known that the fractional Dp-branes can be viewed as ordinary D$(p+2)$-branes wrapped on an integer basis of vanishing two-cycles [2]. In this paper we want to test this correspondence by explicit calculations of string scattering amplitudes. In particular we focus on the fractional Dp-branes in type IIA string theory compactified on $T^4/\mathbf{Z_2}$, where $\mathbf{Z_2}$ is generated by the parity operator on the four compact spatial coordinates. More specifically, we want to check that the boundary action for fractional Dp-branes, obtained geometrically from the one of D$(p+2)$ branes suitably wrapped, is in agreement with the CFT predictions. To do this we first determine the supergravity fields which couple to the fractional D-brane

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by means of the boundary state formalism (for a review see [6]). This simply amounts to obtain the terms of the world-volume action of the fractional D-brane which are linear in the bulk fields. Then, we go on and compute the quadratic terms of the fractional branes world-volume action using the same methods that have been discussed for ordinary D-branes in [7, 8, 9, 10, 11, 12, 13].

These calculations confirm that the world-volume action obtained geometrically is consistent with the explicit conformal calculations.

This paper is organized as follows: in section 2 we review the geometric approach to fractional D-branes and derive accordingly their world-volume action; in section 3 we calculate the linear and quadratic scattering amplitudes between the massless fields that couple to fractional D-branes. The three appendices are devoted respectively to fix our notation, determine the normalization for the boundary state and to list the vertices used in various calculations.

2 The geometric approach to the fractional Dp brane action

The action for type IIA supergravity in ten dimensions can be written (in the string frame) as:

$$
S_{\mathrm{IIA}} = \frac{1}{2\kappa_{10}^2}\left\{ \int d^{10}x\ \sqrt{-\det G}\ \mathrm{e}^{-2\Phi}\ \mathrm{R(G)} + \int \mathrm{e}^{-2\Phi}\Big[\ 4d\Phi \wedge\ ^{*_{10}}d\Phi\ -\frac{1}{2}\mathrm{H}_{(3)}\wedge\ ^{*_{10}}\mathrm{H}_{(3)}\Big]\right.
$$

$$
\left.+\frac{1}{2}\Big[\ \mathrm{F}_{(2)}\wedge\ ^{*_{10}}\mathrm{F}_{(2)}\ +\ \widetilde{\mathrm{F}}_{(4)}\wedge\ ^{*_{10}}\widetilde{\mathrm{F}}_{(4)}\ -\mathrm{B}_{(2)}\wedge\mathrm{F}_{(4)}\wedge\mathrm{F}_{(4)}\ \Big]\right\} \tag{1}
$$

where

$$
\mathrm{H}_{(3)} = d\mathrm{B}_{(2)}\ ,\qquad \mathrm{F}_{(2)} = d\mathrm{C}_{(1)}\ ,\qquad \mathrm{F}_{(4)} = d\mathrm{C}_{(3)} \tag{2}
$$

are respectively the field strengths corresponding to the NS-NS 2-form potential, to the 1-form and 3-form potentials of the R-R sector; moreover:

$$
\widetilde{\mathrm{F}}_{(4)}\ =\ \mathrm{F}_{(4)} - \mathrm{C}_{(1)}\wedge\mathrm{H}_{(3)}\ , \tag{3}
$$

and

$$
\kappa_{10}\ =\ 8\,\pi^{7/2}\,g_s\,\alpha'^2
$$

where g_s is the string coupling constant.

The usual BPS Dp-branes with p even are solutions of the classical field equations that follow from the action (1), which are charged under the R-R $(p+1)$-form potentials and preserve sixteen supercharges. On the other hand, the interaction between the massless fields and a Dp brane is described by the boundary action (also in the string frame):

$$S = -\frac{T_p}{\kappa_{10}} \left\{ \int d^{p+1}\xi \; e^{-\Phi} \sqrt{-\det[\tilde{G} + \tilde{B} + 2\pi\alpha'F]} \right.$$
$$\left. - \int \left[e^{B+2\pi\alpha'F} \wedge \sum_n C_{(n+1)} \right]_{p+1} \right\} . \tag{4}$$

where T_p/κ_{10} is the D-brane tension, $T_p = \sqrt{\pi}(2\pi\sqrt{\alpha'})^{3-p}$ and the tilde denotes the pullback.

Let us now consider type IIA supergravity compactified on the orbifold

$$\mathbb{R}^{1,5} \times T^4/\mathbf{Z_2} \tag{5}$$

where $\mathbf{Z_2}$ is the reflection parity that changes the sign to the four coordinates of the torus T^4, which we take to be x^6, x^7, x^8 and x^9. The bulk action for this theory is still given by (1) suitably compactified [5], but with κ_{10} replaced by $\kappa_{\text{orb}} = \sqrt{2}\kappa_{10}$.

The orbifold (5) is a singular limit of a smooth K3 surface. In this case, besides the usual Dp-branes (bulk branes) which can freely move in the compact directions, there are *fractional* Dp-branes [2] which are instead constrained to stay at one of the orbifold fixed points. The fractional Dp-branes can be viewed as ordinary D$(p+2)$-branes wrapped on an integer basis of cycles $c_I \in \text{H}_2(\text{K3}, \mathbb{Z})$, with a non trivial B-flux, in the collapsing limit in which these cycles shrink [14]. We study this theory at one fixed point, with fractional branes that extend only in the space-time directions. In this context it is enough to consider only one cycle for each fixed point. By thinking of the resolved space in the neighborhood of one fixed point, the geometrical counterparts of the twisted gauge fields [2] are the Kaluza-Klein $p+1$ forms A_{p+1} that come from the harmonic decomposition

$$C_{(p+3)} = C_{p+3} + \sqrt{V_4} \, A_{p+1} \wedge \hat{\omega} \tag{6}$$

where C_{p+3} and A_{p+1} are six-dimensional forms (from now on we will use italic style for the six dimensional fields), while $\hat{\omega}$ is the anti-self-dual (1,1) form, which is Poincarè dual to the relevant cycle c and it's normalized as follows:

$$\int_{\text{K3}} \hat{\omega} \wedge^{\star_4} \hat{\omega} = 1 \qquad \int_c \hat{\omega} = \sqrt{2} \tag{7}$$

The factor of normalization in front of A_{p+1} (V_4 is the volume of the torus T^4) in (6) is due to the fact that generally the compactification works differently for untwisted and twisted fields, but we want to write an homogeneous six-dimensional effective action. Consider, for example, the kinetic term in the ten dimensional Lagrangian (1) for the RR 3-form:

$$\frac{1}{2\kappa_{\text{orb}}^2} \int \frac{1}{2} \, dC_{(3)} \wedge^{*10} dC_{(3)} \ .$$

If we decompose as in (6)

$$C_{(3)} \;=\; C_3 + \sqrt{V_4} A_1 \wedge \hat{\omega}$$

we get a standard term in the six-dimensional effective Lagrangian

$$\frac{1}{2\kappa_6^2} \int \frac{1}{2} \left[\, dC_{(3)} \wedge^{*6} (dC_{(3)}) \; + dA_{(1)} \wedge^{*6} (dA_{(1)}) \right] \tag{8}$$

where $\kappa_6 = \kappa_{\text{orb}}/\sqrt{V_4}$.

Now consider a configuration in which the ten-dimensional two-form B takes a background value different from zero along the cycle c:

$$B_{(2)} \;=\; B_{(2)} + \sqrt{V_4} \, b \, \hat{\omega} \tag{9}$$

and wrap a D$(p+2)$-brane on c. According to what we have said, the resulting configuration will be a fractional Dp-brane and its boundary action can be obtained from (4) using the wrapping formulas (6) and (9). In the Einstein frame this action reads:

$$
\begin{aligned}
S_{\text{boundary}} \;=\; & -\frac{\hat{T}_p}{\sqrt{2}\kappa_6} \left\{ \int d^{p+1}x \; e^{-\frac{1-p}{2}\phi + \frac{1}{2}\sum_i \eta_i} \sqrt{-\det\left[\tilde{G} + e^{-\phi}\tilde{B} + 2\pi\alpha' \, e^{-\phi}F \right]} \right. \\
& \left. - \int \left[e^{B+2\pi\alpha' F} \wedge \sum_n C_{n+1} \right]_{p+1} \right\} \\
& -\frac{N_{T,p}}{\kappa_6} \left\{ \int d^{p+1}x \; e^{-\frac{1-p}{2}\phi + \frac{1}{2}\sum_i \eta_i} \sqrt{-\det\left[\tilde{G} + e^{-\phi}\tilde{B} + 2\pi\alpha' \, e^{-\phi}F \right]} \, \tilde{b} \right. \\
& \left. - \int \left[e^{B+2\pi\alpha' F} \wedge \sum_n \left(C_{n+1}\tilde{b} + A_{n+1} \right) \right]_{p+1} \right\} \tag{10}
\end{aligned}
$$

where:

$$\hat{T}_p \;=\; \frac{T_p}{\sqrt{V_4}} \qquad N_{T,p} \;=\; \frac{\sqrt{2}\,T_p}{4\pi^2\alpha'} \qquad . \tag{11}$$

The six dimensional dilaton field ϕ and the scalar fields η_i are defined as follows

$$\phi \;=\; \Phi - \frac{1}{4}\ln\left(\prod_i G_{ii}\right) \qquad G_{ii} = \mathrm{e}^{2\eta_i} \quad \text{with } i = 6,7,8,9 \tag{12}$$

while the twisted field $\tilde{b}$ represents the fluctuation part of b around the background value characteristic of the $\mathbf{Z}_2$ orbifold [14, 15], which in our notation is

$$b \;=\; \frac{1}{2}\frac{(2\pi\sqrt{\alpha'})^2}{\sqrt{V_4}} \;+\; \tilde{b}\,. \tag{13}$$

We want to stress that what is really characteristic of this background is the flux of the field B along the cycle c. This does not depend on the normalization of the (1,1) form $\hat{\omega}$ and it has to take the value $\frac{\sqrt{2}}{2}(2\pi\sqrt{\alpha'})^2$.

In this paper we want to test the validity of the geometric approach just described by explicitly computing string scattering amplitudes. To do this, we need the string mode fluctuations for canonically normalized fields. In our case they take the form (see also [5]):

$$\begin{aligned}
g_{\mu\nu} &= \eta_{\mu\nu} + 2k_6\,h_{\mu\nu} \\
\phi &= \kappa_6\,\varphi \\
B_{\mu\nu} &= \sqrt{2}\kappa_6\,b_{\mu\nu} \\
C_{(n)} &= \sqrt{2}\kappa_6\,c_n \\
A_{(n)} &= \sqrt{2}\kappa_6\,a_n \\
\tilde{b} &= \sqrt{2}\kappa_6\,\beta \\
\eta_i &= \kappa_6\,\hat{\eta}_i
\end{aligned} \tag{14}$$

In the next paragraphs we examine all the linear and quadratic couplings of the closed strings to the D-brane and compare them with the results we can predict from the action (10), finding complete agreement.

3 String amplitudes for fractional branes

On general grounds the properties of D-branes can be discussed either by studying their interaction with open strings [16] or by introducing the boundary state (for a review see [6]).

The boundary state is a BRST invariant state written in terms of the closed string oscillators. It encodes the couplings of the D-branes with all states of the closed string spectrum and inserts a boundary on the closed string world sheet enforcing on it the appropriate boundary conditions. In an orbifold background like (5) it has four different components which correspond to the (usual) NS-NS and R-R untwisted sectors and to the NS-NS and R-R twisted sectors [15, 17, 18, 19], (for a more detailed analysis see appendix B). Generally it provides two essential information: the couplings with the massless closed string states (from which we read the mass and the charges) and their long distance behaviour (from which we study the space time geometry) [20].

We are going to analyze the first aspect in the following subsection.

3.1 Linear coupling

The generator of the interaction amplitudes of any closed string state with a fractional Dp-brane is the following boundary state:

$$|\mathrm{D}p\rangle \;\; = \;\; N_{U,p}\left(|B\rangle_{NS,U} \pm |B\rangle_{R,U}\right) \pm N_{T,p}\left(|B\rangle_{NS,T} \pm |B\rangle_{R,T}\right) \tag{15}$$

where (see appendix B):

$$N_{U,p} \;\; = \;\; \frac{1}{\sqrt{2}}\frac{\hat{T}_p}{2} \tag{16}$$

$$N_{T,p} \;\; = \;\; \frac{\sqrt{2}\,T_p}{4\pi^2\alpha'}$$

The various amplitudes can be computed by simply saturating $|B\rangle$ with the properly normalized closed string states. For example the twisted NS-NS scalar β, which is related to the fluctuations of the b field of (9), is represented by the following vertex operator:

$$V_\beta^{(-1,-1)}(z,\bar{z}) = \frac{1}{\sqrt{2}}\,\beta\,C_{ab}S^a(z)\bar{S}^b(\bar{z})\,\mathrm{e}^{ik\cdot X(z,\bar{z})}\,\mathrm{e}^{-\phi(z)}\mathrm{e}^{-\tilde{\phi}(\bar{z})}\,\Lambda(z)\tilde{\Lambda}(\bar{z}) \tag{17}$$

We have written it in the (-1,-1) superghost picture, since this is the appropriate picture to use with the boundary state [20]. Then the coupling between β and a fractional Dp-brane will be given by:

$$\langle\, V_\beta \mid \mathrm{D}p\,\rangle \;\;=\;\; -\sqrt{2}\; N_{T,p}\; V_{p+1}\; \beta \tag{18}$$

We see that this coupling is in perfect agreement with what we get from the action (10). We can now proceed analogously for all other massless states that can couple to the D-brane. Here we simply list the results (see appendix C for the explicit expressions of the vertices), since the explicit calculations are identical to those already discussed in the literature (see e.g. [20]):

$$\begin{aligned}
\langle\, V_h \mid \mathrm{D}p\,\rangle \;\;&=\;\; -\frac{\hat{T}_p}{\sqrt{2}}\; V_{p+1}\; \mathrm{Tr}(h{\cdot}L)\\[2mm]
\langle\, V_{\hat{\eta}_i} \mid \mathrm{D}p\,\rangle \;\;&=\;\; -\frac{\hat{T}_p}{2\sqrt{2}}\; V_{p+1}\; \hat{\eta}_i\\[2mm]
\langle\, V_{a_{p+1}} \mid \mathrm{D}p\,\rangle \;\;&=\;\; \sqrt{2}N_{T,p}\; V_{p+1}\; a_{p+1}\\[2mm]
\langle\, V_{c_{p+1}} \mid \mathrm{D}p\,\rangle \;\;&=\;\; \hat{T}_p\; V_{p+1}\; c_{p+1}
\end{aligned} \tag{19}$$

where the fields $h_{\mu\nu}$, $\hat{\eta}_i$, a_{p+1} and c_{p+1} have been defined in (14), and the longitudinal matrix L is given by

$$L = \frac{1}{2}\,(1+S) \tag{20}$$

where S is

$$S^\mu{}_\nu \;\;=\;\; \begin{pmatrix} \mathbf{1}_{p+1} & 0 \\ 0 & -\mathbf{1}_{5-p} \end{pmatrix} \tag{21}$$

and encodes the Neumann and Dirichlet boundary conditions. For later convenience we also introduce the matrix N

$$N = \frac{1}{2}\,(1-S) \tag{22}$$

with components only in the directions normal to the brane.

It is easy to check that the couplings (19) are in agreement with those prescribed by the action (10).

3.2 Quadratic couplings

There are two equivalent ways to reconstruct the quadratic couplings of the closed string states with a Dp-brane from the conformal theory. One approach is to use the boundary state, sew it with a string propagator and saturate it with the vertices of the two external states S_1 and S_2. This approach has been used in [7] to obtain the anomalous coupling of a D-brane with the $B_{\mu\nu}$ field and also non-anomalous couplings with curvature terms [8]. The other approach is instead to evaluate the correlator of the vertices on the disk. This latter method has been discussed in [10, 11, 12] for ordinary D-branes of type II and extended in [13] for branes of type 0.

In the boundary states approach, the amplitude describing the coupling of two closed string states is given by [1]:

$$A_{S1,S2} = C_0^{sp} \hat{N}^3 \langle V_{S_1} \mid V_{S_2}(1,1) \, \Delta \mid \mathrm{D}p \rangle \tag{23}$$

where

$$C_0^{sp} = \tfrac{4\pi}{\alpha'} \left(\tfrac{\pi}{\kappa_6}\right)^2 \quad , \quad \hat{N} = \tfrac{1}{\pi} \, \kappa_6 \tag{24}$$

are the topological normalization respectively for the sphere and for the states [21], and Δ is the usual closed string propagator:

$$\Delta = \frac{\alpha'}{4\pi} \int_{|z|<1} \frac{d^2 z}{|z|^2} z^{L_0} \bar{z}^{\tilde{L}_0} \tag{25}$$

The overlap equations defining $|\mathrm{D}p\rangle$ allow us to identify the right movers with the left movers by means of the $S^\mu{}_\nu$ matrix (21) encoding the information about the longitudinal and the transverse directions. After this identification the right movers operators appear only in the boundary exponential and annihilate the left or right vacua (for details see e.g. [9]).

The other approach is to evaluate the correlator of the vertices on the disk [10, 11]. Here one can use the *doubling trick* to duplicate the region inside the disk to fill the whole complex plane. This is done introducing the matrix S (21). The conformal invariance of the disk allows us to fix three (real) parameters for the four punctures and leaves just one modulus to be integrated over. In this way we get the right correlator for the vertices, but we do not determine the right normalization involving the tension of the brane. Perfect equivalence between the two approaches is reached by adding in the latter case an overall multiplication

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[1] The projective invariance of the sphere has allowed us to fix the position of the S_2 vertex at $z = \bar{z} = 1$.
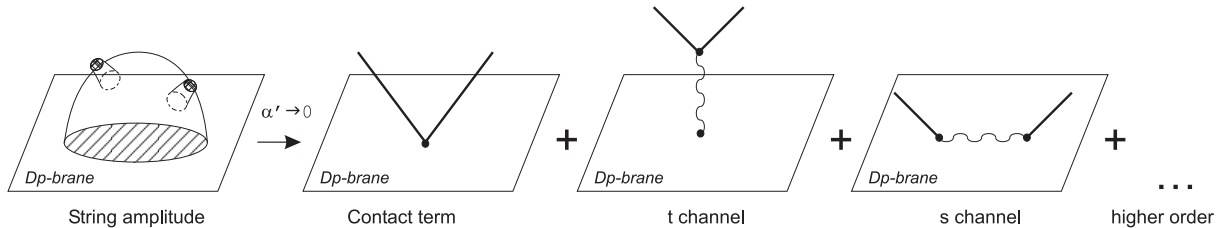
Figure 1: The low energy expansion. The first term in the expansion represent the contact term; this is directly the two point vertex present in the effective Lagrangian. However, the two point string function contains also the other two exchange diagrams.

factor $\kappa_6\ N_{U,p}$ or $\kappa_6\ N_{T,p}$ (see appendices B and C for our conventions) according to the required boundary sector (15).

In order to get the interaction vertices of the boundary action, we must perform the low energy limit $\alpha' \to 0$. It is crucial to realize that the string amplitudes in this limit exhibit two types of divergences which are due to the exchange of massless particles in the s- and t-channel (fig. 1). So, we must evaluate these diagrams and subtract them from the string amplitudes.

To evaluate the t-channel exchange diagram (fig. 1), we need the S_{IIA} bulk action (1) suitably compactified [5]. In fact we evaluate it with the sewing technique: we start from the couplings of the boundary with an intermediate state, propagate it in the bulk and finally sew with a three point interaction vertex read from the compactified bulk action. Of course, the exact cancellation of the divergences requires the sum on all possible intermediate states.

The s-channel diagram represents the interaction of the external states with an open string state living on the brane. In this case, the two interaction points can be obtained by string theory techniques because now they are irreducible. So, for each external states we correlate its vertex with an intermediate open string vertex, and then sew together with the proper propagator.

In ten dimensions this technique has been successfully applied in [12] to reproduce the world volume action (4). We want to extend it to the orbifold compactification in order to reproduce the action describing the world volume interactions of the fractional branes (10). In this case there is a larger variety of states and we summarize in table 1 all possible quadratic couplings. For shortness and notation convenience we introduce a 4×4 matrix Q according to table 1, to easily identify the sectors we will refer to. For example the symbol Q_{23} means a string amplitude between a closed string of the R-R untwisted sector, a closed

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	NS,U	R,U	NS,T	R,T
NS,U	B_{NSU}	B_{RU}	B_{NST}	B_{RT}
R,U		B_{NSU}	B_{RT}	B_{NST}
N,U			B_{NSU}	B_{RU}
R,T				B_{NSU}

Table 1: Here we summarize all the possible quadratic couplings. In the columns there are the possible sector for particle 1; the rows are for particle 2. For each couple of states we report the only interacting sector of the Dp-brane.

string state of the NS-NS twisted sector and the boundary state; its explicit expression is:

$$Q_{23} = C_0^{sp}\hat{N}^3\,\kappa_6 N_{T,p}\,\langle\,V_{NST}^{(0,-1)}\mid V_{RU}^{(-1/2,-1/2)}(1,1)\,\Delta\mid B_{RT}^{(-3/2,-1/2)}\,\rangle \qquad (26)$$

Each element of this matrix Q must reproduce some of the terms of the boundary action. So, for example, we have:

$$Q_{11} \rightarrow e^{-\frac{1-p}{2}\phi}\sqrt{-\det(\tilde{G}+e^{-\phi}\tilde{B})}\Big|_{\substack{\text{quadratic}\\ \text{coupling}}}$$

$$Q_{12} \rightarrow C_{p-1}\wedge B$$

$$Q_{22} \rightarrow 0$$

The calculation of these terms and their correspondence with the appropriate boundary action has been discussed respectively in [12], [7] and [10] for the case of a flat background in ten dimensions. Here we have performed the same calculation in the case of the orbifold compactification and, apart from some obvious changes, the same structure is recovered. We will concentrate here on the amplitudes involving one particle in a twisted sector, which are specific of the fractional branes; we consider:

$$
\begin{aligned}
Q_{13} &= C_0^{sp}\hat{N}^3\,\kappa_6 N_{T,p}\,\langle\,V_{NST}^{(-1,-1)}\mid V_{NSU}^{(0,0)}(1,1)\,\Delta\mid B_{NST}^{(-1,-1)}\,\rangle\\
Q_{14} &= C_0^{sp}\hat{N}^3\,\kappa_6 N_{T,p}\,\langle\,V_{RT}^{(-1/2,-3/2)}\mid V_{NSU}^{(0,0)}(1,1)\,\Delta\mid B_{RT}^{(-3/2,-1/2)}\,\rangle\\
Q_{23} &= C_0^{sp}\hat{N}^3\,\kappa_6 N_{T,p}\,\langle\,V_{NST}^{(0,-1)}\mid V_{RU}^{(-1/2,-1/2)}(1,1)\,\Delta\mid B_{RT}^{(-3/2,-1/2)}\,\rangle\\
Q_{24} &= C_0^{sp}\hat{N}^3\,\kappa_6 N_{T,p}\,\langle\,V_{RT}^{(-1/2,-1/2)}\mid V_{RU}^{(-1/2,-1/2)}(1,1)\,\Delta\mid B_{NST}^{(-1,-1)}\,\rangle
\end{aligned}
\qquad (27)
$$

The explicit expressions for the vertices are given in appendix C.

For example, for the amplitude Q_{13} we get:

$$Q_{13} = \alpha' \, \kappa_6 N_{T,p} \, \frac{\Gamma(-\alpha't/2)\Gamma(2\alpha'q^2)}{\Gamma(1-\alpha't/2+2\alpha'q^2)} \, \sqrt{2} \left(2q^2 \, a_1 + \frac{t}{2} \, a_2 \right) \tag{28}$$

where

$$\begin{aligned}
t &= -(p_1+p_2)^2 = -2p_1 \cdot p_2 \\
q^2 &= p_1 \cdot L \cdot p_1 = \frac{1}{2} p_1 \cdot S \cdot p_1
\end{aligned} \tag{29}$$

are respectively the momentum transferred to the brane and the longitudinal one.

The kinematic factors in (28) are:

$$\begin{aligned}
a_1 &= \beta \; p_2 \cdot \varepsilon_1 \cdot p_2 \\
a_2 &= \beta \left[2(q^2 - t/4) \, \mathrm{Tr}(\varepsilon_1 \cdot S) + p_1 \cdot S \cdot \varepsilon_1 \cdot p_2 + p_2 \cdot \varepsilon_1 \cdot S \cdot p_1 + p_1 \cdot S \cdot \varepsilon_1 \cdot S \cdot p_2 \right]
\end{aligned}$$

where ε_1 is the polarization of the first state in the NS-NS untwisted sector. So by choosing the appropriate ε_1 we select the graviton, Kalb-Ramond or dilaton field.

In the low energy limit $\alpha' \to 0$, we note the amplitude is divergent:

$$Q_{13} = -\alpha' \, \kappa_6 N_{T,p} \, \sqrt{2} \left(\frac{2}{t} a_1 + \frac{1}{2q^2} a_2 \right) \tag{30}$$

As explained before, we must subtract these divergences owing to the field theory diagrams. Let's start with the t-channel. For definiteness we will consider the interaction between $h_{\mu\nu}$ and β. The only three point interaction vertex we can read from the bulk action is the kinetic term of the scalar β:

$$\frac{1}{2} \left(2 \, \kappa_6 \, h^{\mu\nu} \right) \partial_\mu \beta \, \partial_\nu \beta \tag{31}$$

Now we must sew the corresponding field theory amplitude $A^{bulk} = -2\kappa_6 \, \varepsilon_1^{\mu\nu} \, p_\mu^2 \, \beta_2 \, p_\nu^3 \, \beta_3$ with the propagator $-\frac{1}{t}$ for the twisted scalar and finally with the linear coupling with the brane (18):

$$A_t(h, \beta) = -2 \, p_2 \cdot \varepsilon_1 \cdot p_2 \, \beta_2 \, \frac{1}{t} \, \kappa_6 \, \sqrt{2} N_{T,p} \tag{32}$$

Next, let's consider the s-channel. We need to evaluate the interaction between the external states and an intermediate open string state λ living and propagating on the brane.

These λ^i fields represent the excitations of the transverse coordinates X^i, whose expectation values correspond to the brane position; they appear only in the pull back of the metric $\tilde{G}$. With our normalization for the boundary action (10), from the kinetic term we can read the propagator:

$$\langle \, \lambda^i \lambda^j \, \rangle \;\; = \;\; - \, \frac{\sqrt{2} \, \kappa_6}{\hat{T}_p} \, \frac{1}{p^2} \, N^{ij} \tag{33}$$

Now, instead of evaluating the interaction from the string, we will show an equivalent and shorter way [12]. The idea is to imagine the interaction of a generic state as coming from the boundary action interpreted as the first order of a Taylor series in the transverse coordinates. Considering the field β as in the example, we can read the coupling from the next to linear order of (18):

$$V_{\beta,\lambda} \;\; = \;\; \sqrt{2} \, N_{T,p} \, V_{p+1} \, \lambda^j \partial_j \beta \tag{34}$$

Analogously, the interaction between h and λ gets two contributes:

$$V_{h,\lambda} \;\; = \;\; \frac{\hat{T}_p}{\sqrt{2}} \, \left[\, \mathrm{Tr}(\varepsilon_1 \cdot L) \, p_1 \cdot N \cdot \lambda \; - \; 2 \, p_1 \cdot L \cdot \varepsilon_1 \cdot N \cdot \lambda \,\right] . \tag{35}$$

The first is the next to linear order of the Taylor expansion and the second appears directly in the pull back $2 \, h_{ia} \partial^a X^i$. Sewing all together we get:

$$A_s(h,\beta) \;\; = \;\; - \, \kappa_6 N_{T,p} \sqrt{2} \, \frac{1}{q^2} \, \beta \, \left[\, p_2 \cdot N \cdot p_1 \, \mathrm{Tr}(\varepsilon_1 \cdot L) \; - \; 2 \, p_2 \cdot N \cdot \varepsilon_1 \cdot L \cdot p_1 \,\right] \tag{36}$$

Using the relation (20,22) and (29), we can subtract the divergent part of (30) in all the amplitudes. In terms of the *standard* fields (14) directly appearing in the action (10), we obtain:

$$Q_{13}(\phi,\tilde{b}) - A_t(\phi,\tilde{b}) - A_s(\phi,\tilde{b}) \;\; = \;\; - \, \frac{N_{T,p}}{\kappa_6} \, \frac{p-1}{2} \phi \, \tilde{b}$$

$$Q_{13}(h,\tilde{b}) - A_t(h,\tilde{b}) - A_s(h,\tilde{b}) \;\; = \;\; - \frac{N_{T,p}}{\kappa_6} \, \mathrm{Tr}\left(\kappa_6 \, h \cdot L\right) \tilde{b}$$

$$Q_{13}(B,\tilde{b}) - A_t(B,\tilde{b}) - A_s(B,\tilde{b}) \;\; = \;\; 0$$

$$Q_{13}(\hat{\eta}_a,\tilde{b}) - A_t(\hat{\eta}_a,\tilde{b}) - A_s(\hat{\eta}_a,\tilde{b}) \;\; = \;\; - \frac{N_{T,p}}{\kappa_6} \, \frac{1}{2} \hat{\eta}_a \, \tilde{b}$$

Similar calculations for the other massless closed string modes yield:

$$Q_{14} = \frac{N_{T,p}}{\kappa_6} \frac{1}{2\,(p-1)!}\, A_{\mu_1\ldots\mu_{p-1}}\, B_{\mu_p\mu_{p+1}}\, \epsilon^{\mu_1\ldots\mu_{p+1}} \tag{37}$$

$$Q_{23} = \frac{N_{T,p}}{\kappa_6} \frac{1}{(p+1)!}\, \tilde{b}\, C_{\mu_1\ldots\mu_{p+1}}\, \epsilon^{\mu_1\ldots\mu_{p+1}} \tag{38}$$

These are exactly the couplings we expect from the action (10).

4 Conclusions

By directly examining various string scattering amplitudes, we have extracted various world-volume interactions for the effective field theory on fractional D-branes. Our results are in perfect agreement with the Born-Infeld and Wess-Zumino terms of the action (10), numerical coefficients included. A further check could be to evaluate the quadratic coupling involving two twisted states explicitly.

As already emphasized, the techniques we have used are rather general. In particular it could be interesting to apply them to the case of fractional branes extended also in the compact directions. There are indeed many reasons to argue that the resulting space time configurations are extended objects with a smaller density of mass and charges.

Acknowledgement

A Notation

In the orbifold background (5) the Lorentz group $SO(1,9)$ is naturally broken to $SO(1,5)\otimes SO(4)$. We reserve the Latin capital letters for $SO(1,9)$ indices, the Greek ones for the six-dimensional spacetime and the Latin ones for the compact space.

We have then, for the ten dimensional vector representation:

$$
X^M \rightarrow \{X^\mu, X^i\} \qquad \begin{pmatrix} M &=& 0\ldots 9 \\ \mu &=& 0\ldots 5 \\ i &=& 6\ldots 9 \end{pmatrix} \tag{39}
$$

and for the spinorial representation [2] :

$$
\begin{array}{rcl}
S^A &\rightarrow& \{S^\alpha \otimes S^a\} \\
S^{\dot A} &\rightarrow& \{S^{\dot\alpha} \otimes S^a\}
\end{array}
\qquad
\begin{pmatrix}
A, \dot A &=& 1\ldots 32 \\
\alpha, \dot\alpha &=& 1\ldots 8 \\
a &=& 1\ldots 4
\end{pmatrix}
\tag{40}
$$

B   Boundary normalization

In each sector of the theory we can construct the boundary state:

$$
|B, k, \eta\rangle = e^{i\theta} \exp\left(\sum_{l>0} \left[\frac{1}{l} \alpha^\mu_{-l} S_{\mu\nu} \tilde\alpha^\nu_l \right] + i\eta \sum_{m>0} \left[\psi^\mu_{-m} S_{\mu\nu} \tilde\psi^\nu_m \right] \right)
$$
$$
|B, k, \eta\rangle^{(0)} \tag{41}
$$

where l and m are integer or half-integer depending on the sector, k denotes the momentum of the ground state and θ is a phase equal to π in the untwisted R-R and twisted NS-NS sectors, to $3\pi/2$ in the untwisted NS-NS sector and to 0 in the twisted R-R sectors. The parameter $\eta = \pm 1$ describes the two different spin structures [22, 23], and the matrix S (21) encodes the boundary conditions of the Dp-brane which we shall always take to be diagonal. We omit always the ghost and superghost part of the boundary states, which are as in the usual case [20, 24]. In order to obtain a localized Dp-brane (say in $y = 0$), we have to take the Fourier transform of the above boundary state, where we integrate over the directions transverse to the brane. For the untwisted sector we obtain:

$$
|B, y = 0, \eta\rangle = \prod_{\nu=6}^{9} \left(\sum_{n_\nu} e^{i \frac{n_\nu}{R_\nu} \hat q} \right) \frac{1}{(2\pi)^{5-p}} \int dk^{5-p} \, e^{ik\hat q} \, |B, k, \eta\rangle, \tag{42}
$$

while for the twisted one:

$$
|B, y = 0, \eta\rangle = \frac{1}{(2\pi)^{5-p}} \int dk^{5-p} \, e^{ik\hat q} \, |B, k, \eta\rangle. \tag{43}
$$

---

[2] We recall the orbifold projection selects just one chirality for $SO(4)$.

The invariance of the boundary state under the GSO and the orbifold projection always requires that the physical boundary state is a linear combination of the two states corresponding to $\eta = \pm$. In the conventions of [20, 24] these linear combinations are of the form:

$$|B\rangle_{NS,U} = \frac{1}{2}\Big(|B,+\rangle_{NS,U} - |B,-\rangle_{NS,U}\Big) \tag{44}$$

$$|B\rangle_{R,U} = \frac{1}{2}\Big(|B,+\rangle_{R,U} + |B,-\rangle_{R,U}\Big) \tag{45}$$

$$|B\rangle_{NS,T} = \frac{1}{2}\Big(|B,+\rangle_{NS,T} + |B,-\rangle_{NS,T}\Big) \tag{46}$$

$$|B\rangle_{R,T} = \frac{1}{2}\Big(|B,+\rangle_{R,T} + |B,-\rangle_{R,T}\Big) \tag{47}$$

$$\tag{48}$$

A fractional Dp brane state can be written as:

$$|\mathrm{D}p\rangle = N_{U,p}\Big(|B\rangle_{NS,U} \pm |B\rangle_{R,U}\Big) \pm N_{T,p}\Big(|B\rangle_{NS,T} \pm |B\rangle_{R,T}\Big) \tag{49}$$

Now we want to determine the constant $N_{T,U}$ solving the open-closed consistency condition. We have to compare the closed string cylinder diagram with the open string one-loop diagram. We obtain:

$$N_{U,p} = \frac{1}{2\sqrt{2}}\hat{T}_p \tag{50}$$

$$N_{T,p} = \sqrt{2\pi}(4\pi^2\alpha')^{\frac{1-p}{2}} \tag{51}$$

where

$$\hat{T}_p = \frac{T_p}{\sqrt{V_4}} \quad \text{and} \quad T_p = \sqrt{\pi}(4\pi^2\alpha')^{\frac{3-p}{2}} \tag{52}$$

C   Vertices

According to the notation established in appendix (A), we write the following vertices:

NS-NS Untwisted

$$V_{NSU}^{(0,0)}(z,\bar{z}) \;=\; \varepsilon_{MN}\,[\partial X^M + i(k\cdot\psi)\psi^M]_z[\partial\tilde{X}^N + i(k\cdot\tilde{\psi})\tilde{\psi}^N]_{\bar{z}}\,U^{(0,0)}(z,\bar{z}) \quad (53)$$

where

$$
\begin{array}{llll}
h \;:\; & \varepsilon_{\mu\nu}^{h} = +\,\varepsilon_{\nu\mu}^{h} & \varepsilon_{ij}^{h} = 0 \\
B \;:\; & \varepsilon_{\mu\nu}^{B} = -\,\varepsilon_{\nu\mu}^{B} & \varepsilon_{ij}^{B} = 0 \\
\varphi \;:\; & \varepsilon_{\mu\nu}^{\varphi} = \frac{1}{\sqrt{6-2}}\,(\eta_{\mu\nu} - p_\mu l_\nu - l_\mu p_\nu) & \varepsilon_{ij}^{h} = 0 \\
\eta_k \;:\; & \varepsilon_{\mu\nu}^{\eta_k} = 0 & \varepsilon_{ij}^{\eta_k} = \delta_i^k \delta_j^k
\end{array}
$$

NS-NS Twisted

$$V_{\beta}^{(-1,-1)}(z,\bar{z}) \;=\; \frac{1}{\sqrt{2}}C_{ab}S^a(z)\bar{S}^b(\bar{z})\,T^{(-1,-1)}(z,\bar{z}) \quad (54)$$

R-R Untwisted

$$V_{RU}^{(-1/2,-1/2)}(z,\bar{z}) \;=\; F_{A\dot{B}}S^A(z)\bar{S}^{\dot{B}}(\bar{z})\,U^{(-\frac{1}{2},-\frac{1}{2})}(z,\bar{z}) \quad (55)$$

where

$$F_{A\dot{B}} = \sum_{n\;even}\frac{1}{n!}\,(C\Gamma^{\mu_1\cdots\mu_n})_{A\dot{B}}\,F_{\mu_1\cdots\mu_n}$$

R-R Twisted

$$V_{RT}^{(-1/2,-1/2)}(z,\bar{z}) \;=\; G_{A\dot{B}}S^\alpha(z)\bar{S}^{\dot{\beta}}(\bar{z})\,T^{(-\frac{1}{2},-\frac{1}{2})}(z,\bar{z}) \quad (56)$$

where

$$G_{\alpha\dot{\beta}} = \sum_{n\;even}\frac{1}{n!}\,(C\Gamma^{\mu_1\cdots\mu_n})_{\alpha\dot{\beta}}\,G_{\mu_1\cdots\mu_n}$$

and

$$U^{p,q}(z,\bar{z}) \;=\; \mathrm{e}^{p\phi}\mathrm{e}^{q\tilde{\phi}}\,\mathrm{e}^{ik\cdot X(z,\bar{z})}$$

$$T^{p,q}(z,\bar{z}) \;=\; \mathrm{e}^{p\phi}\mathrm{e}^{q\tilde{\phi}}\,\Lambda(z)\,\tilde{\Lambda}(\bar{z})\,\mathrm{e}^{ik\cdot X(z,\bar{z})}$$

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